**Modular Arithmetic and Number Theory**

Modular arithmetic is used for many applications. A primary application is with time and how we compute what time it will be a certain number of hours from now, keeping in mind that every 24 hours, time repeats.

1. Do research on modular arithmetic. Explain what it is in a summary paragraph.

2. Show how you would use modular arithmetic to answer these questions:

a. What month will it be 83 months from now? (Now = May)

b. If today is Wednesday, what day will it be 152 days from now?

c. A pilot is scheduled to fly for 5 consecutive days and rest for three consecutive days. If today is the second day of her rest shift, determine whether she will be flying:

* 60 days from today
* 90 days from today
* 240 days from today
* Was she flying 6 days ago?
* Was she flying 20 days ago?

3. The procedure of casting out nines can be used to check arithmetic problems. The procedure is based on the fact that any whole number is congruent to the sum of its digits in modulo 9. For example, the number 5783 and 5 + 7 + 8 + 3, or 23, both have a remainder of 5 when divided by 9 and are therefore both congruent to 5 (mod 9): .

a. Explain why this procedure works. If your explanation is in the form of a formal proof, all the better.

b. Perform the indicated operation and check your solution by casting out nines.

5321

+4786

4. In arithmetic and algebra, the statement, “If , then or . That is, for the product of two numbers to be 0, at least one of the factors must be 0. Can the product of two nonzero numbers equal 0 in a specific modulo system? If so, in what type of modulo system can this result occur?

a. Construct multiplication tables for mod. systems 3-9

b. Which, if any, of the multiplication tables in part a have products equal to 0 when neither factor is 0?

c. Which, if any, of the multiplication tables in part a have products equal to 0 only when at least one factor is 0?

d. Using the results in parts b and c, write a conjecture as to which modulo systems have a product of 0 when neither factor is 0.